Introduction to Statistics

Joao Lourenço (joao.lourenco@sib.swiss) and Rachel Marcone (rachel.marcone@sib.swiss)

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Parametric and non parametric tests

T-tests: summary

T-test in general Used to compare means

One-sample t-test Compare the mean of a sample to a given number
Two-sample t-test Compare the means of two samples
Paired t-test Compare the difference between pairs of related data points

Assumptions for the t-test

- This test is one of the most widely used tests. However, it can be used only if the background assumptions are satisfied.
  - Data values must be **independent**.
  - Data in each group must be obtained via a **random sample** from the population.
  - The variances for the two independent groups are **equal**.
  - Data in each group are **normally** distributed.
  - Data values are **continuous**.

T-test in practice

- Analysts generally don't use the standard (Student) t-test.

- The Welch t-test takes into account different sample variances.

- If the variances are actually the same, it provides the same results as the standard Student t-test.

```
> t.test(KO_WT$weight ~ KO_WT$genotype)


                    Welch Two Sample t-test

data:  KO_WT$weight by KO_WT$genotype
t = -1.4261, df = 18.905, p-value = 0.1702
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -3.0078465  0.5705536
sample estimates:
mean in group KO mean in group WT
       30.32366         31.54231
```
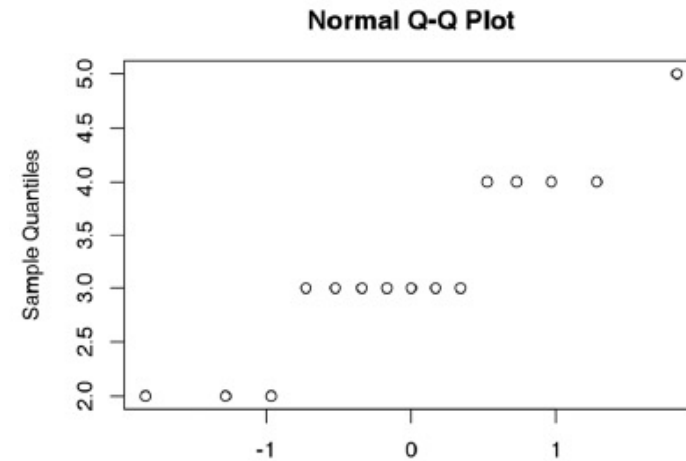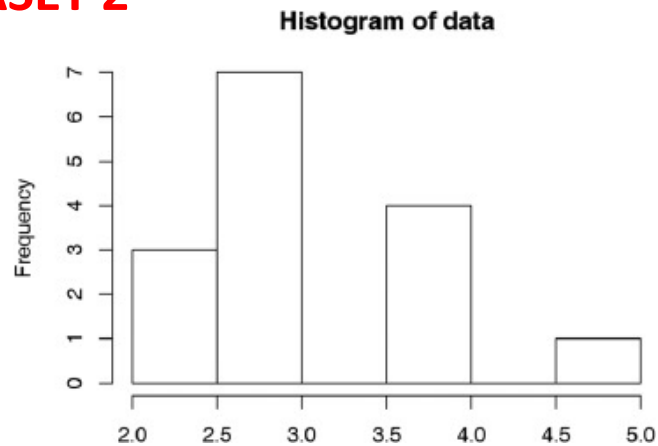
Graphical tools to assess normality

DATASET 1
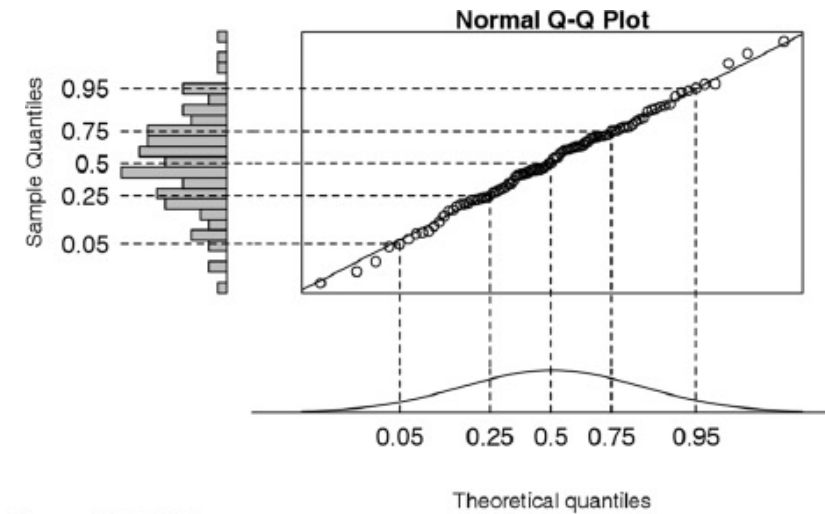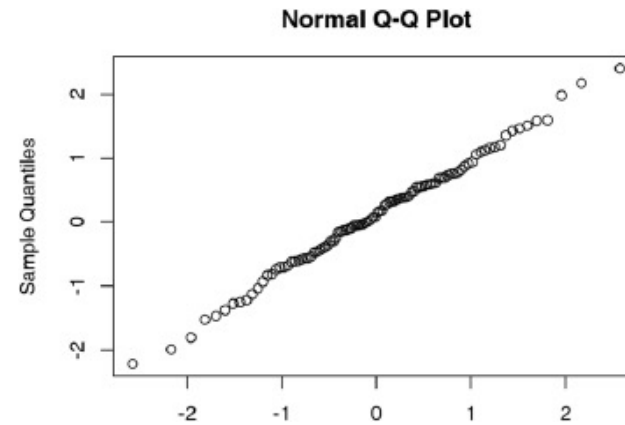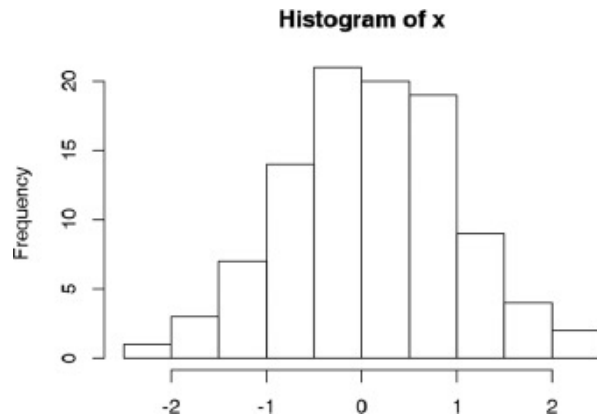
DATASET 2

Normality tests

- Examples: Shapiro-Wilk, Kolmogorov-Smirnov.

```
> shapiro_test(weight$weight)
# A tibble: 1 x 3
  variable        statistic  p.value
  <chr>               <dbl>    <dbl>
1 weight$weight       0.902    0.166
```

- Normality tests are statistical tests
  - They test **against** normality (and can not show/prove normality)
  - They may not have enough power
  - Conversely, if *n* is large, even a very small departure from normality may be significant.

What happens if the data is not normally distributed ?

- Transform the data using the log function
  - Examples of data that should be logged:
    - Highly asymetrical data
    - Data spanning several orders of magnitude
    - Data originating from ratios

- Another possible solution: non-parametric tests
  - Wilcoxon rank-sum test (WRS)
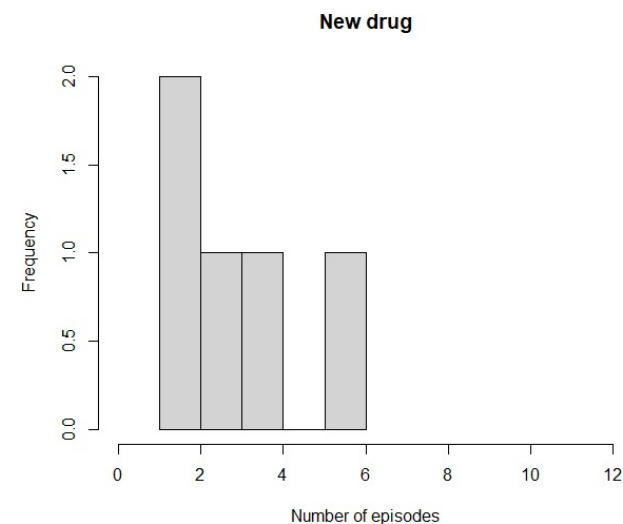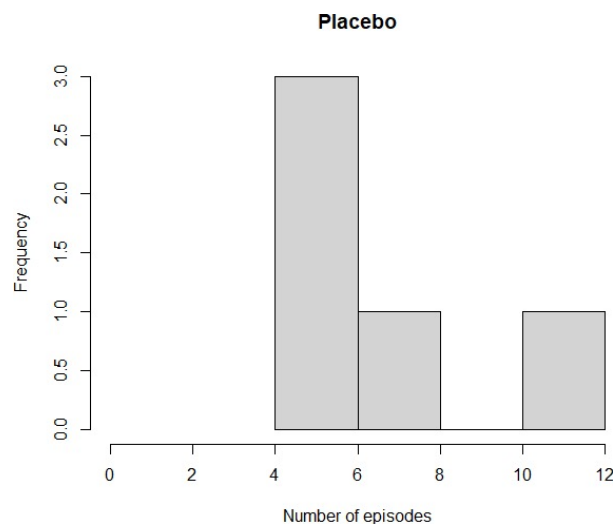 - Mann–Whitney U test
  - Mann–Whitney–Wilcoxon (MWW)
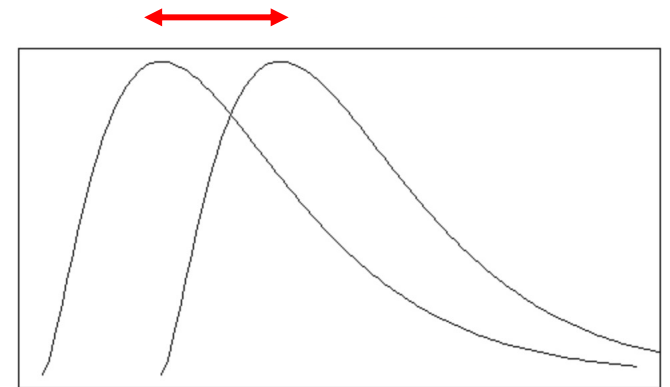
Mann–Whitney U test

- General formulation:
  - All the observations from both groups are independent of each other
  - The variable is ordinal or numerical
  - H_0: the distributions of both populations are equal
  - H_1: the distributions are not equal

Mann–Whitney U test: an example

Consider a Phase II clinical trial designed to investigate the effectiveness of a new drug to reduce symptoms of asthma in children. Participants are asked to record the number of episodes of shortness of breath over a 1 week period following receipt of either the new drug or a placebo. Is the difference significant ?

Placebo	7	5	6	4	12
New drug	3	6	4	2	1

Mann–Whitney U test: an example

Placebo	New drug	Total sample (ordered)		Ranks	
Placebo	New drug	Placebo	New drug	Placebo	New drug
7	3		1		1
5	6		2		2
6	4		3		3
4	2	4	4	4.5	4.5
12	1	5		6	
		6	6	7.5	7.5
		7		9	
		12		10	

R1 = 4.5+6+7.5+9+10 = 37

R2 = 1+2+3+4.5+7.5= 18

$$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1 = 5\,(5) + \frac{5(6)}{2} - 37 = 3$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2 = 5\,(5) + \frac{5(6)}{2} - 18 = 22$$

Test statistic for the Mann Whitney U Test is denoted **U** =min(U_1,U_2)

Mann–Whitney U test: an example

n1\n2	2	3	4	5	6	7	8	9	10
2							0	0	0
3			0	1	1	2	2	3
4			0	1	2	3	4	4	5
5		0	1	2	3	5	6	7	8
6		1	2	3	5	6	8	10	11
7		1	3	5	6	8	10	12	14
8	0	2	4	6	8	10	13	15	17
9	0	2	4	7	10	12	15	17	20
10	0	3	5	8	11	14	17	20	23

Decision for the test: if U < U_{crit} we reject H0

We do not reject H0 because 3 > 2.

We do not have statistically significant evidence at α =0.05, to show that the two populations of numbers of episodes of shortness of breath are not equal.

Low power because of small sample sizes ?

```
> t.test(placebo,newdrug)

        Welch Two Sample t-test

data:  placebo and newdrug

t = 2.199, df = 6.6639, p-value = 0.06574

alternative hypothesis: true difference in means is not equal to 0
```

Take home message

- How to handle (non-)normality
  - Start by making sure that your data is of the right type
  - If the data spans several orders of magnitude, is made of ratios or is very asymmetrical, consider taking the log.
  - If n is large (>20 or 30), then you should not have to worry.
  - As long as your data is not "too" far from normality (and you are careful with borderline p-values), the t-test will return a meaning full value.

- How to handle (non-)normality: alternatively
  - Perform both a t-test and a Wilcoxon rank-sum test
  - If they return a coherent result, it is stronger that if you performed only one test.
  - If the results are incoherent, look at the data to find out what produced this.