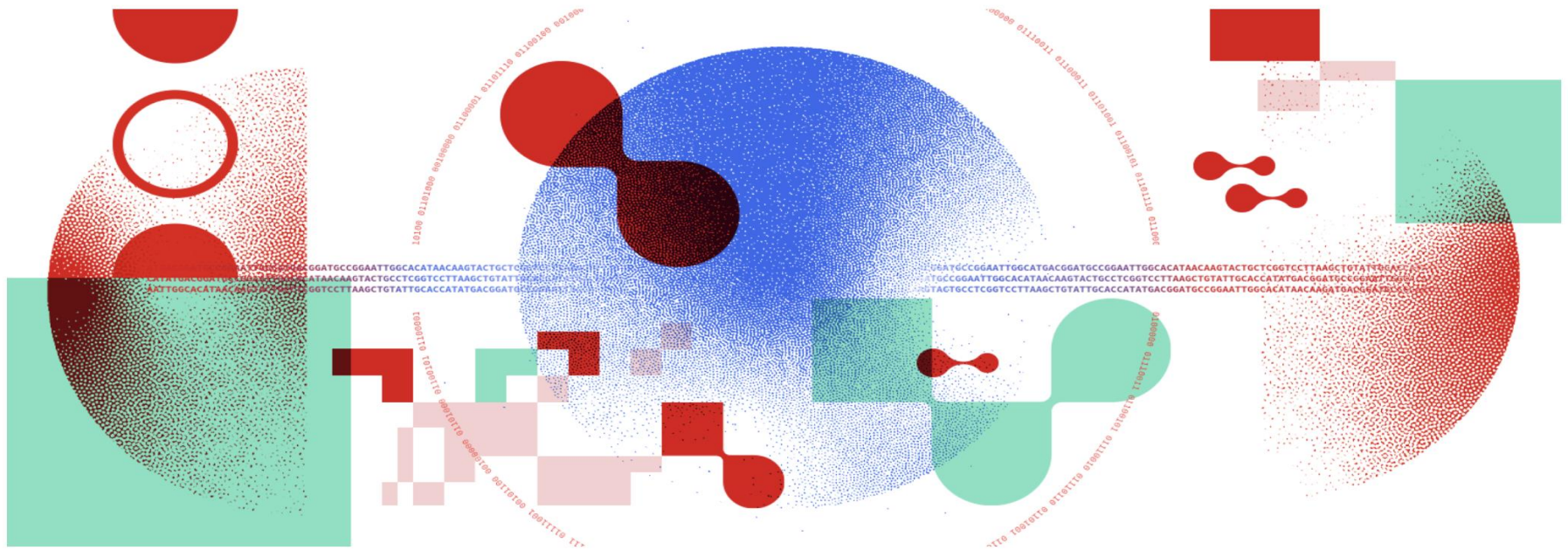




Advanced Statistical Modelling

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Generalized Additive Models



Generalized Additive Models (GAMs)

- So far, we have seen a number of approaches for flexibly predicting a response Y on the basis of a single predictor X
- Here we explore the problem of predicting Y on the basis of several predictor $X_1, X_2, \dots, X_p$
- GAMs provide a general framework for extending a standard linear model by allowing smooth functions of each of the variables, while maintaining additivity
 - The response can be either quantitative or qualitative

Generalized Additive Models (GAMs)

- A natural way to extend the multivariable linear regression model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \varepsilon_i$$

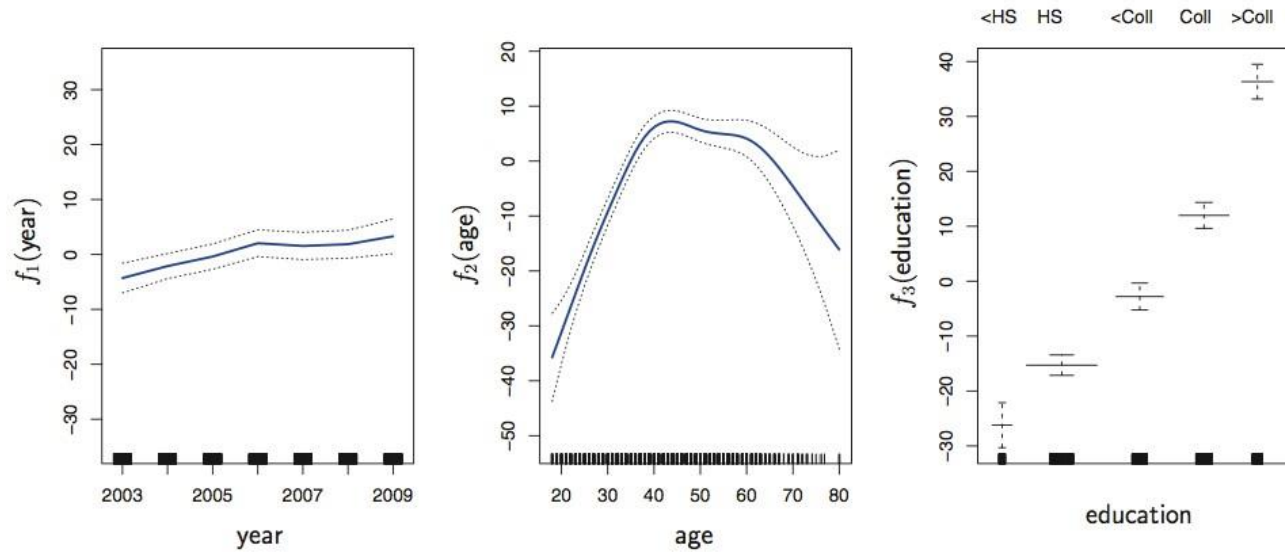
in order to allow for smooth relationships between each feature and the response is to replace each linear component with a smooth function:

$$y_i = \beta_0 + f_1(x_{i1}) + f_2(x_{i2}) + \dots + f_p(x_{ip}) + \varepsilon_i$$

- This is an example of a GAM
- It is called additive model because we calculate a separate f_j for each X_j , and then add together all of their contributions
- The beauty of GAMs is that we can use various smoothing methods as building blocks for fitting an additive model
 - Spline regression
 - Smoothing splines
 - Local regression (loess)

$$\text{wage} = \beta_0 + f_1(\text{year}) + f_2(\text{age}) + f_3(\text{education}) + \epsilon$$

smoothing
splines



Fitting GAMs

$$\text{wage} = \beta_0 + f_1(\text{year}) + f_2(\text{age}) + f_3(\text{education}) + \epsilon$$

- To fit a GAM using smoothing splines and local regression

```
gam(wage ~ s(year, df = 5) + lo(age, span = .5) + education)
```

- Coefficients not that interesting; fitted functions are
- Can mix terms (linear or nonlinear) and use `anova(...)` to compare models

Fitting GAMs

- The gam package in R uses an approach known as backfitting
 - Involves repeated updating of the fit for each predictor while holding others fixed
 - Each time we update a function, we simply apply the fitting method for that variable to a partial residual
- A partial residual for X_3 for example, has the form $r_i = y_i - f_1(x_{i1}) - f_2(x_{i2})$ and therefore If we know f_1 and f_2 then we can fit f_3 by treating this residual as a response in a smooth regression on X_3
- The mgcv package in R uses mixed modeling framework for smoothing

Fitting GAMs

- GAMs allow us to fit a smooth f_j to each X_j , so that we can automatically model non-linear relationships that standard linear regression will miss
 - This means we do not need to manually try many different transformations on each variable independently
- The smooth fits can potentially make more accurate predictions for the response Y
- Because the model is additive, we can still examine the effect of each X_j on Y individually while holding all of the other variables fixed

Smoothing Exercise: The “wage” data

- Mid-Atlantic Wage Data
 - Wage and other data for a group of 3000 workers in the Mid-Atlantic region

References

- Semiparametric Regression; by David Ruppert, M.P. Wand, and R.J. Carroll; Cambridge University Press
- Generalized Additive Models; by T.J. Hastie and R.J. Tibshirani; Chapman & Hall/CRC
- Generalized Additive Models - An Introduction with R (2nd Edition); by Simon N. Wood; CRC Press